

TW3421x - Week 6 - CreditMetrics

Hi there welcome to Week 6.

In this week, we will consider our survey of models for the estimation, for the assessment of the probability of default of a counterparty.

We will consider a model like CreditMetrics, which is an evolution - we could say - of Merton's model, but we will also consider other approaches, such as for example the approach of Credit Risk Plus.

So... let's start with CreditMetrics.

CreditMetrics was introduced in 1997 by J.P. Morgan & Co, now JP Morgan Chase.

It is a structural model of default, and it is worth noticing that it also takes into account the risk of credit deterioration. Do you remember the second component of credit risk?

From a mathematical point of view, CreditMetrics can be seen as a Merton-like model.

The intuition behind CreditMetrics is rather simple: instead of using liabilities as thresholds on which we define the default event, we will use credit rating. We will define the thresholds for the model using credit rating.

We consider a company that has been assigned to some credit rating class (say B) at the beginning of period $[0, T]$. This means that, for this company, we also have information about the transition probabilities to other rating classes and, naturally, about the probability of default by the end of the period.

These transition probabilities can be computed using historical data, or other techniques.

Do you remember how to read a transition matrix like the one you see on your screen?

If the answer is no, go back to Week 4.

With p_{ij} , we indicate the probability that our company will be in rating class j at time T .

To simplify notation, we order the j 's, so that j equal to 0 means "default" and j equal to n means that our company is in the best rating class, e.g. AAA. This implies that we consider n rating classes, including default.

With p_{i0} , we clearly denote the probability of default of the company at time T .

Using the same notation we used in Merton's model, with V_T we indicate the value of the assets of the company at time T .

If you remember, under Merton's model, we have said that V_T is linked to a normal distribution; and we have used the CDF of a Normal to compute the PD under Merton.

We can then define a set of threshold values d_0 , d_1 , ..., d_{n+1} , such that d_0 is equal to minus infinity and d_{n+1} is equal to plus infinity.

For all the other d 's, we choose them so that the probability that V_T is between d_j and d_{j+1} is equal to p_j .

In this way, we translate the transition probabilities into a series of thresholds for the asset value of the company.

By construction d_1 is the default threshold. If V_T is smaller than d_1 in T , our company defaults.

All the other thresholds simply define the other rating classes, so that our company belongs to class j if V_T is between d_j and d_{j+1} .

In CreditMetrics, in order to exploit the properties of the standard Normal distribution, the asset value V_T is transformed into a quantity X_T using the formula you see on your screen. For statisticians, this is nothing more than a standardization.

Naturally, we do the same for the threshold values.

The consequence of this transformation is that now our company belongs to class j if X_T is between d_j^* and d_{j+1}^* .

Let's consider an example to better understand the mechanism thanks to which we transform transition probabilities into thresholds for X_T .

Consider a B-rated company over a 1-year period. And assume we have the following fictitious Moody's transition matrix.

The probabilities we are interested in are those in the green row.

Let X_B be the rescaled asset value, according to CreditMetrics, of our B-rated company at the end of the year. According to the data in the problem, we have that $T=1$.

As anticipated, it can be shown that X_B is distributed as a standard Normal.

Using the table, the probability that our B-rated company will improve its rating to Aaa by the end of the year is 0.01% or 0.0001.

For the Aa class we have 0.0004.

If we consider the probability of default, this is 0.0473, and so on of all the other probabilities and classes.

Using the quantile function of a standard Gaussian, we will now compute the threshold values we need for the CreditMetrics model.

Let's start from the lowest rating class, that is default. The probability of default is 0.0473. This probability, under the standard Normal quantile function, corresponds to a value of -1.6716.

If X_B is smaller than/equal to -1.6716, we have that our B-rated company defaults by the end of the year. Notice that -1.6716 is simply the d1 star threshold.

How can we compute the quantiles? Clearly with R, using the `qnorm` function.

If X_B is bigger than -1.6716, but smaller than or equal to -1.6009, our company is in the Ca-C class.

The value -1.6009 is computed using again the quantile function. But now notice that we have to cumulate probabilities. Starting from the lowest class, that is default, we also add the probability of moving to class Ca-C.

Class Caa is then defined by thresholds -1.6009 and -1.1790. This last value is again obtained by inverting the cumulative probability. Just look at the computation I did with `qnorm`.

Finally, the Aaa class is reached if X_B is larger than 3.7190, which is the 99.99% quantile. Notice that 99.99% is equal to $1-0.0001$, which is the probability for a B-rated company to enter the Aaa class.

Therefore, by ordering the rating classes, cumulating their transition probabilities and applying the quantile function, we can compute all the thresholds we are interested in.

And now a question for you: what is the rating class, at the end of the year, of our B-rated company, if $X_B=1.31$?

Naturally, the complete CreditMetrics model also takes into account dependence among defaults.

This is done using probabilistic tools called Copulas. In CreditMetrics a Gaussian copula is typically used, according to the so-called Li's model.

But, as you can imagine, such a topic requires a knowledge of probability which I cannot assume in this course.

In R there is a very nice package, that you can use to play with CreditMetrics. It is called the CreditMetrics package.

It's a rather complex package for the knowledge we have of R, but I am posting the link to the package on the course platform, so that - if you want - you can play with it.

So... for today we are over. See you next time. Goodbye.