Text Lecture 6a – The critical Mach number

This lecture is about the effect of compressibility on the performance of airfoils. More in particular we will focus on a flow condition for a Mach number called the critical Mach number.

We know by now that the velocity on the suction side of the airfoil of a wing at a moderate angle of attack is higher than the speed of the incoming flow. As an example here you can see an airfoil moving through air with a Mach number of 0.3 at a certain angle of attack. The flow velocity over the airfoil suction side increases and at some point the maximum velocity -or the highest local Mach number- (or in other terms the minimum pressure coefficient $C_{p,min}$) is reached. In this example, at this angle, this is for a Mach number of 0.44 and a Cp of around -1.7.

If we now increase the flow speed also the Mach number at this point will go up. At Mach=0.5 it will have reached a value of 0.78. And at Mach is 0.6 for the first time the flow over the airfoil suction side will reach the speed of sound and will go sonic. This Mach number of 0.6 is called the critical Mach number. The associated minimum pressure coefficient is called the critical pressure coefficient. Please note that this is just an example. Another airfoil with a different shape at a different angle of attack will likely have a different critical Mach number and at a different location on the airfoil. A comparable graph can be made for one airfoil at different lift coefficients.

Now let us look at the influence of the shape of an airfoil and more in particular at the impact of the airfoil thickness.

Generally, increasing the upper surface thickness will result in higher local velocities, closer to the speed of sound. We know that the Cp distribution more or less shows the local overspeed squared, so higher upper surface velocities mean more negative Cp's

So, at the same flight speed these thicker airfoils will have higher local Mach numbers on the suction side of the airfoil and consequently they will have a lower critical Mach number.

This is shown in the following graph. For a flat plate, with no speed-up over the surface the critical Mach number is equal to the free stream Mach number, so $M_{critical}$ is 1. If we go up in thickness the critical Mach number goes down as is shown by the points on the curve.

Also shown is the effect of compressibility according to the Prandtl-Glauert correction, so with increasing Mach number the Cp and obviously also the minimum Cp goes to more negative values. For each of the four examples the critical Cp has a different value.

There is a relation between the critical Cp on an airfoil and the critical Mach number that connects the points on the Cp-lines. How can we derive it?

Okay, let us first write the cp a little differently. The definition of cp is p – p infinity divided by q infity. But we can also write this as p infnity divided by q infinity times p divided by p infinity – 1. Now let us concentrate a bit here on this q infinity. So q infinity is ½ rho v infinity squared. And this v infinity squared can also be written as M infinity squared times a infinity squared. So it follows that q infinity is ½ rho infinity M infinity squared times a infinity squared. Now we also have a relation of a with R and T, and p and rho. So a infinity squared is gamma times p infinity divided by rho infinity. And if we combine these two, then we have q = ½ M infinity squared gamma p infinity. So we have cp is p infinity divided by q infinity times p divided p infinity minus one, and if we fill in for this q the relation we just found we get cp = 2 p infinity divided by M infinity squared gamma p infinity times p divided by p infinity minus one. Now p infinity is in the numerator and the denominator so we have cp is 2 divided by M infity squared gamma times p divided by p infinity minus one. But also for this p divided by p infinity we can use the second version of the isentropic relations. If we write p_t divided by p = 1+gamma-1 divided by 2 M squared to the power gamma divided by gamma minus one, this p_t if we bring the flow isentropically to rest, we have the total pressure like in the stagnation point of a wing, but we can also write p_t divided by p_infinity is one + gamma minus one divided by two M infinity squared to the power of gamma divided by gamma minus one. Now if we divide those two then we have p divided by p infinity is one plus a half gamma minus one M infinity squared divided by one plus a half gamma minus one M squared to the power gamma divided by gamma minus one. Now this we can insert in the equation for Cp. Cp=2 divided by gamma M infinity squared times one plus a half gamma minus one M infinity squared divided by one plus a half gamma minus one M squared to the power gamma divided by gamma minus one, minus one.

This is a general relation between the C_p and the Mach number at a specific location on the airfoil contour and the free stream Mach number.

For M=1 the Mach number is the critical Mach number and the Cp is the critical pressure coefficient.

So finally this is the relation for the critical pressure coefficient and the critical Mach number. And if we now go back to the graph we see that the points on the Cp lines are connected with this new relation.

We found a general expression for the critical Mach number and the critical Cp and we have an airfoil dependent relation for the Cp. The intersection gives us the critical Mach number for our airfoil.

Now what happens if we have reached the critical Mach number and we would still increase the flow speed?

Locally a region with supersonic flow will start to develop, which will spread to the leading edge with still increasing Mach number and -since also on the lower side of the airfoil locally the velocities are higher than the free stream velocity- also the lower surface will show pockets of supersonic flow. At some point shocks will appear. Over a shock wave the pressure increases. So the flow will see an adverse pressure gradient, which will get bigger with increasing shock severity. The boundary layer will separate when the shock is big enough, creating a high pressure drag. The Mach number at which this rise in drag starts, is called the Mach number for ***drag divergence***.

If we further increase the Mach number, the drag rises rapidly until the free stream Mach number has reached the value of one and the flow over the entire airfoil is supersonic. This rapid drag rise due to the forming of shocks was the reason that initially people thought that there was no end to this drag rise and they called it the sound barrier. However, as you can see in this picture, when the flow over the airfoil is entirely supersonic it reduces again due to a combination of shock and expansion waves. You simply need to generate enough thrust overcome this high drag rise.

The conclusion is that – to avoid heavy shocks on the suction side of the airfoil for a specific lift coefficient– we need to restrict the upper surface thickness. However, we also need strength and stiffness in a wing, which requires enough building height. And we need sufficiently high flight Mach numbers. There is a way to accomplish this, without the high drag associated with upper surface shocks: supercritical airfoils.

These airfoils have supersonic velocity over a large part of the upper surface, but slightly above the critical Cp so that heavy shocks will be avoided. That is why they are relatively flat on the suction side. The thickness is now shifted to the lower side.

To compensate for the loss of lift due to the restricted upper surface Cp some aft-loading has been introduced at the pressure side, which gives the typical s-shape of the lower surface.

This slide gives a good example of the difference between a conventional airfoil and a supercritical airfoil at high subsonic speed. Both generate the same lift, but the supercritical airfoil only has weak shocks and a much lower drag.

In the next lecture, we will concentrate on the difference in performance of two-dimensional airfoils and finite wings.