

AE1110x - Lecture 3b - The boundary layer on a flat plate

In this video we will learn more about laminar and turbulent flow.

Let's investigate the flow over a very thin flat plate at zero angle of attack. More in particular we are interested in the drag of the plate.

Isaac Newton already formulated an equation for the shear stress:

The shear stress τ is μ times the velocity gradient.

μ is the dynamic viscosity coefficient or in short viscosity

So for the shear stress on the surface of a flat plate we are looking at the velocity gradient near the wall at $y=0$. Here you see the boundary layer velocity profile with the velocity u varying from zero at the surface, to the undisturbed free stream velocity V at the edge of the boundary layer.

Let us define the local Reynolds number: it is defined as ρ times v times x divided by μ , with x running along the flat plate starting at the leading edge.

When a fluid starts to flow over the flat plate it begins to form a laminar boundary layer. The streamlines are smooth and regular. The flow moves in neat layers.

At some distance from the leading edge the character of the flow has changed into irregular, random and chaotic movements. This is a turbulent boundary layer

The transition process from laminar to turbulent flow is a complicated mechanism of growing disturbances in the flow. We will talk about that later.

First of all let us look at the development of the boundary layer thickness along the plate.

With his boundary layer theory, Prandtl, together with his former student Blasius, was able to simplify the Navier-Stokes equations in such a way that they could get analytical results for a laminar boundary layer. From this theory we find that the local boundary layer thickness at a station x from the leading edge is equal to 5.2 times x divided by the square root of the local Reynolds number.

So, starting at the leading edge of the plate the boundary layer in fact develops parabolically, with the square root of x

Now let's look at a flat plate with length L and a width of 1 m. At distance x from the leading edge we have a small strip of the plate with length dx in flow direction.

The total force on the entire plate is the total pressure force plus the total friction force. Since the plate is flat and very thin and under zero pressure gradient, we have no pressure forces.

The friction force on the element dx is τ times the area, which is dx times 1.

The total skin friction drag on the plate is The integral from the leading edge to the trailing edge, so from $x=0$ to $x=L$ of τ times dx

From laminar boundary layer theory we can find that the local skin friction coefficient, defined as the shear stress over the undisturbed dynamic pressure is equal to 0.664 divided by the squareroot of the local Reynolds number. So, the skin friction coefficient decreases with increasing distance from the leading edge.

To calculate the entire aerodynamic force we must integrate.

Before we do that, let us first have another look at the Reynolds number. The Reynolds number based on a length L was defined as ρ times V times L , divided by μ , right? Well, the density and the dynamic viscosity (or just viscosity) are both defined by the same quantities of the fluid, for instance temperature and pressure. So we can define the kinematic viscosity, ν , by the viscosity μ , over the density ρ . Then we can simplify the Reynolds number to V times L over ν .

At a low Reynolds number we have viscous flow and the effect of friction in the flow is high. It decreases with increasing Reynolds number. Now back to the flat plate.

So D_f , the friction force on the flat plate, is the integral from zero to the length L

of C_{fx} times q_0 times dx ; since this was τ_w .

We know that C_{fx} is 0.664 divided by the square root of Re_x .

Together this forms the integral from zero to L of 0.664 times q_0 times dx

divided by the square root of Re_x .

We assume that q_∞ is constant, so D_f is 0.664 times q_0 times the integral from zero to L of dx divided by the square root of Re_x .

Now Re_x has a component of x . We can write that as the integral of dx divided by the square root of x , but then we have to divide the other part by the square root of the undisturbed velocity divided by the undisturbed kinematic viscosity.

The integral from dx divided by the square root of x is the integral from

x to the power -0.5 and this is two square root x .

So we have D_f is 0.664 times q_∞ divided by the square root of V_∞ divided by the kinematic viscosity times 2 times the square root of L .

And this is 1.328 times q_∞ times L divided by the square root of V_∞ times

L divided by the kinematic viscosity.

Now, you may recognize here that what is under the square root here is of course something

that has to do with the Reynolds number.

So, it is in fact 1.328 times q_{∞} times L divided by the square root of Re_L , so the Reynolds number based on the flat plate length.

Now let us define the skin friction drag coefficient.

C_f is the drag force divided by q_{∞} times S .

Now, this is D_f divided by q_{∞} times L times 1 , because we have a width of the flat plate of $1m$.

Now, if we write C_f with the result of the drag force calculation, then we get:

$C_f = 1.328$ divided by the square root of Re_L .

So we have found the skin friction coefficient on a flat plate is equal to 1.328 over the squareroot of the Reynolds number based on the flat plate length L . This equation is only valid for low speed, incompressible flow, and it is reasonably accurate for high-speed subsonic flow.

Unfortunately there is no exact solution for the turbulent boundary layer

But experiments showed that the boundary layer is much thicker than for the laminar case and that the skin friction coefficient inversely varies with the Reynolds number to the power of 0.2

So if we put the two together we see that the friction coefficient in a turbulent flow is much higher than in a laminar flow.

A turbulent flat plate will have a much higher total skin friction drag.