TW3421x - Week 6 - Credit Risk Plus

Hi there, welcome.

In this class, we discuss a new model, which is a called Credit Risk plus.

CR+ was introduced in 1997 by Credit Suisse.

And this is a very interesting model. It's a rather complex model from a mathematical point of view, because it relies on some quite difficult results in actuarial mathematics.

So, in this class we will just give an idea of the model. We cannot enter into much detail for an introductory course like this.

So…let's start.

Suppose that a financial institution has n loans of a given type.

For simplicity, we will consider these loans to be homogeneous in terms of risk, so we can say that the 1-year PD of each loan is p. In other terms, it is like we are considering all the loans to be part of the same rating class, say BBB.

For what concerns probability p, it can be naturally obtained from external or internal credit ratings, depending on our preferences.

Let mu be the expected number of defaults in our portfolio of loans. We naturally have that mu is equal to n times p.

I would like you to notice that, in this framework, given the assumptions we have just considered, the probability of observing m different defaults, in a set of n loans, corresponds to the probability of observing m heads, when we toss a coin n times.

p will be the probability of observing a head. Obviously in a fair coin this probability is 1/2.

In our case, we can assume the coin to be biased, so that p is different…is not 1/2.

But the mechanism is exactly the same.

The probability we have just described can be expressed using the formula you see on your screen.

Do you recognize the Binomial distribution?

A further step in the Credit Risk Plus model is to assume p, the probability of default of each counterparty, to be small, while the total number of counterparties, that is n, is large. This means that we have a large portfolio of loans.

Given these assumptions, the Binomial distribution is well approximated by another discrete distribution: the Poisson distribution.

The probability of observing m defaults is now e to the power of -mu times mu to the power of m, over m factorial.

The probability of observing m defaults can then be combined with historical information about the probability distribution for the losses experienced when a certain type of counterparty defaults.

This leads us to the computation of a probability distribution for the total losses from defaults.

On that distribution we can then compute quantities such as VaR and ES. And, from those quantities, we can further obtain our capital requirements, according to our A-IRB methodology.

The everyday business life and empirical studies show that the distribution of losses is typically skewed with a long fat right tail, indicating that the probability of observing large losses, extreme losses, is not that small.

The simple approach we have just seen is just an unrealistic version of CR+.

The model actually used by banks is much more complex from a mathematical point of view, because it introduces more realistic components.

For example, in such a model, we do no longer assume defaults to be independent. And we do not consider just risk-homogeneous counterparties, but rather a portfolio made up of loans and securities with different levels of risk, belonging to different rating classes. And we can introduce some additional macroeconomic factors in order to take into account the economic trends and cycles.

As you can imagine, from a mathematical point of view, the complexity of the model rapidly increases.

A nice feature of a model like CR+ is the possibility of getting closed-form analytical results, even for rather complex versions of the model; obviously under specific configurations of the parameters.

If we prefer to have a very realistic model, in which we cannot impose certain conditions on the parameters, we lose closed-form results, but nevertheless we can approach the model through simulations and computational techniques; obtaining, in any case, very useful results in terms of credit risk assessment.

If you are interested in more details, you can find the original credit risk plus document on the course platform.

See you next time.