TW3421x - Week 6 - C-VaR and F-IRB Capital Requirements

Hi there.

Are you ready to see how we can use the PD of a counterparty to actually compute the capital requirements for credit risk for that counterparty?

And are you ready to see how we can compute capital requirements for an entire portfolio of obligors, with which we have some sort of credit relation?

Ok, the topic of today is the computation of capital requirements starting from the probability of default.

We will do that under the IRB approach, and in particular the F-IRB. Because, if you remember, this is the approach in which we start from the PD, and we plug this information into some specific formulas, the risk-weight functions, that will provide us with the desired results in terms of capital requirements.

In this course, we will not consider in detail the A-IRB approach, because the complexity of that approach is too big for a course of this level.

Nevertheless, in the next class, we will consider Credit Risk Plus that can be seen as an A-IRB approach, even if we will just see that from a superficial point of view.

Ok, so, let's start. Let's see how we can compute the capital requirements for credit risk, starting from the PD. Let's go.

Assume that a bank has a large number of counterparties.

Every obligor i has a 1-year probability of default equal to PDi. We have estimated this probability using some models like Merton, Moody's KMV or some manipulations of credit ratings.

Assume that the correlation between each pair of obligors is rho.

In the following we discuss the 1-year time horizon, but all the computations can be easily adapted to other time spans.

A key quantity for the computation of capital requirements under the IRB approaches is the so-called worst case default rate. This is defined as the 99.9% quantile of the default rate distribution. The default rate distribution is typically computed using one-factor copula models, a class of models we do not consider in this course. Heuristically these models give us a way of computing the distribution of the default rates we can expect for groups of counterparties.

The computation of WCDR is performed using the formula you see on your screen.

I do agree with you if you think that this formula has somehow fallen from the sky. That's true: in this course this formula has fallen from the sky.

The point is that, in order to derive this formula completely and formally, we would need - all - a big probabilistic apparatus that we do not have, such as for example the copula model. So, that's it: just take this formula for granted.

Just notice that in the formula you see on your screen for the computation of the WCDR, we use the standard Gaussian distribution. We have Phi, which is the CDF of a standard Gaussian, and we have Phi to the minus 1, which is the quantile function of a standard Gaussian. And as you can see the probability of default is one of the arguments of this function.

As usual, those guys in Basel seem to like the Gaussian distribution a lot. And…obviously in the literature many scholars do not agree with that, but you know…that's life.

In Week 3 we have studied value-at-risk, and we have said that in the credit risk framework, VaR is often called C-VaR.

If we have the exact distribution of credit losses, C-VaR is computed as usual, according to the techniques we have seen together. It is nothing more than a quantile of the loss distribution.

However, when we think about future credit losses, their distribution is not exactly known in advance.

We can surely use past information as a starting point, and then use some additional techniques.

If we assume that all the n obligors of a bank have the same pairwise correlation rho, or we know that the pairwise correlations are very similar, so that we can average them, it can be shown that, for the entire portfolio, the 1-year 99.9% VaR is well approximated by the formula you see on your screen. That formula includes quantities we know, such as the exposure at default and the loss given default, for each obligor i.

Those quantities are usually defined using historical data, empirical studies or, for certain types of instruments, they are provided by the regulator.

For what concerns the expected loss in the portfolio, this is simply given by the sum of all the expected losses for each counterparty, weighted by their probabilities of default.

Do you remember what we have said about the IRB approaches in Week 2?

Do you remember risk factors and risk-weight functions?

We are now ready to understand a little more about that!

If you cannot remember something, go back and check.

Let's consider corporate, sovereign and bank exposures.

In the F-IRB approach, we are free to compute the PD as we prefer, but all the other quantities, or the way in which we have to compute them, is given by the regulator.

Here we are: on your screen you see how we are supposed to compute the correlation parameter for corporate, sovereign and bank exposures.

Have you noticed that in this formula, based on some empirical studies, the PD and the correlation move in two opposite directions?

If the PD increases, correlation decreases. Why?

The idea is that if a company becomes less creditworthy, its PD increases and its probability of default becomes more idiosyncratic, company-specific, and less affected by the overall market conditions.

For corporate, sovereign and bank exposures, the formula used for computing the capital requirements for each counterparty is the one you see on your screen. This is a risk-weight function.

The element MA is called maturity adjustment and it is defined as follows.

Do you see? Once again the formula is given by the regulator.

The maturity adjustment is meant to allow for the fact that, if an obligation lasts longer than 1-year, there is a 1-year credit exposure arising from a possible decline in the creditworthiness of the counterparty, as well as from a possible default of the counterparty.

Just consider that for M, maturity, equal to 1, the maturity adjustment is 1 as well.

The total capital requirements for credit risk in a portfolio are simply given by summing the capital requirements of all the counterparties that belong to the portfolio.

Those capital requirements need to be covered with Tier 1, additional Tier 1 and Tier 2 capital, according to the rules we have seen in Week 1.

Since capital requirements are 8% of RWA, we have that RWA are 12.5 times the amount of capital requirements.

Let's consider an exercise.

For simplicity, we assume risk-homogeneous counterparties. When we have different types of counterparties, computations are only a little bit more tedious, but the idea remains the same.

Suppose that the assets of a bank include 100 million pounds of loans to A-rated corporations.

The PD of all these corporations is estimated to be 0.1%.

Historical data give an average LGD equal to 60%.

The maturity for all loans is 2.5 years.

We want to compute RWA and capital requirements.

Using the formulas we have seen so far, we compute rho, b, the maturity adjustment and the WCDR.

For all these computations we can obviously use R.

RWA are then easily obtained by substituting all values in the RWA formula.

Notice that we can also do the opposite: first compute capital requirements, and then multiply them by 12.5, in order to get RWA.

It is interesting to notice that, under the STA approach, RWA and capital requirements would be higher. You can verify that yourself, using the information we have seen in Week 2. Otherwise, you can simply trust me.

For what concerns the other types of exposures, such as for example retail exposures, the mechanism, the philosophy to obtain RWA and capital requirements is always the same.

We will use different formulas, such as for example those you can see on your screen, but the mechanism is always the same: we have the PD that we have computed using the way we prefer, and then we plug in this information into our formulas, into our risk-weight functions, and we get the desired results in terms of RWA and capital requirements. So, now you are experts in the computation of capital requirements under the IRB approach.

So, this is it for today. See you next time. Goodbye.