GWC-4B Runoff Generation

CTB3300WCx: Introduction to Water and Climate

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Flood recession curve

Terms used

- Depletion curve
- Base flow
- Dry weather flow

Recession flow is groundwater seepage

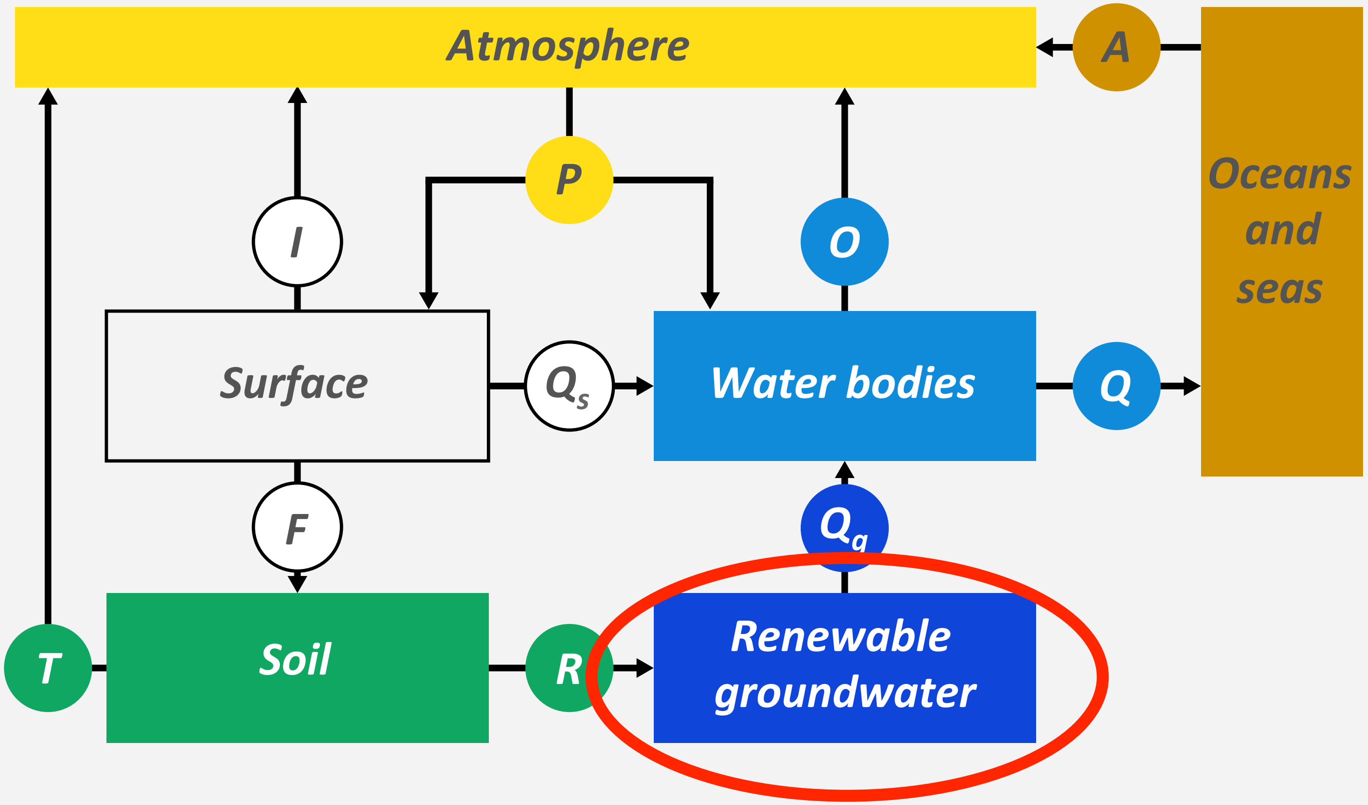
Water balance of the renewable groundwater

$$\left(\frac{\mathrm{d}S_g}{\mathrm{d}t} - R + Q_g\right)_{DB} = 0$$

During the dry season

- $Q = Q_g$
- $R = 0$
- $S = S_g$

Global water resources



Simplified water balance equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -Q$$

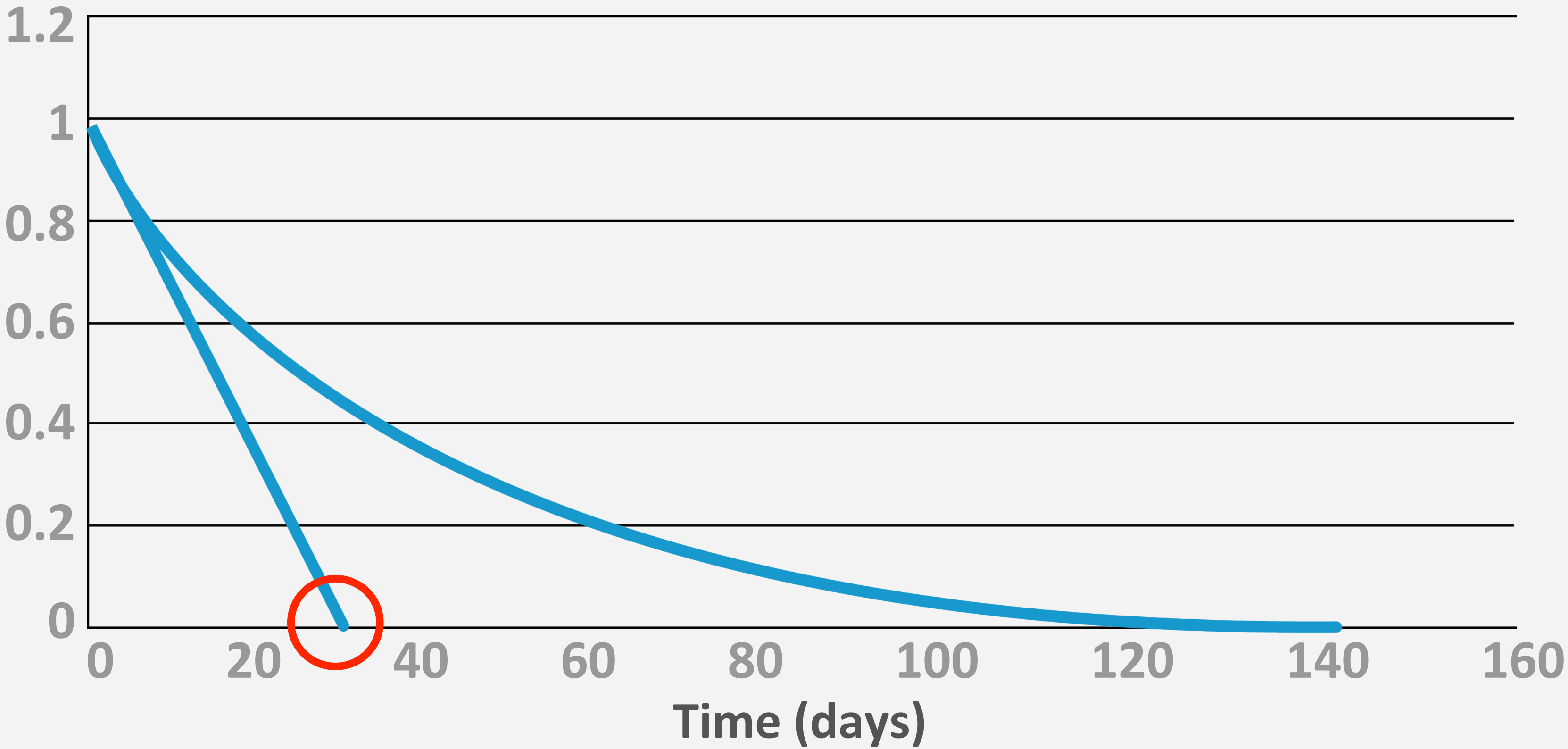
Linear reservoir

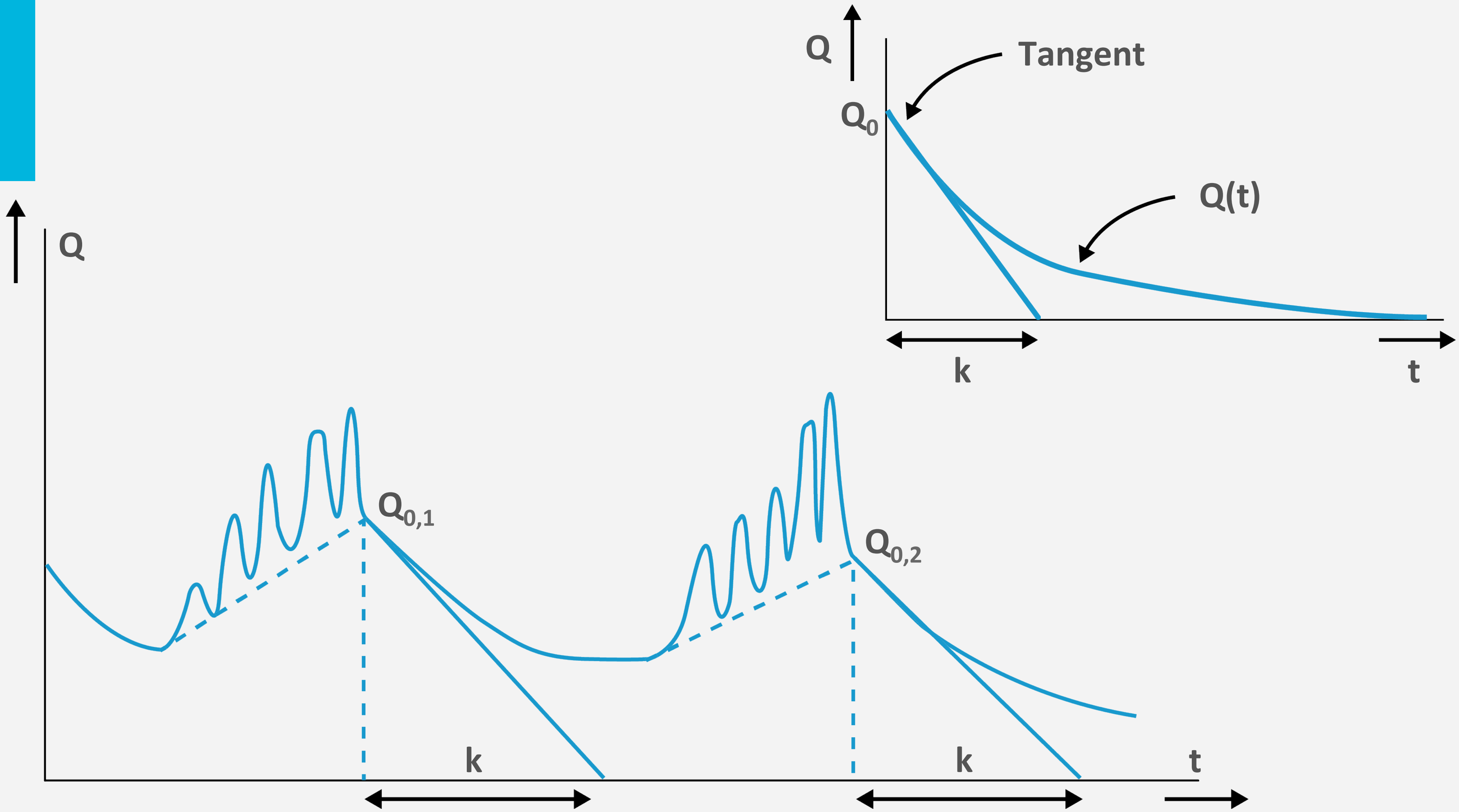
$$S = kQ$$

Solution

$$Q_t = Q_0 \exp\left(-\frac{t}{k}\right)$$

Residence time (k=30 days)




Q
Q
Tangent
Q_0
Q(t)
k
t
Q_{0,1}
Q_{0,2}
k
k
t

Special properties of $Q_t = Q_0 \exp\left(-\frac{t}{k}\right)$

Derivative of the exponential function

$$\frac{\mathrm{d}Q_t}{\mathrm{d}t} = -\frac{1}{k} Q_0 \exp\left(-\frac{t}{k}\right) = -\frac{Q_t}{k}$$

Integral of the exponential function

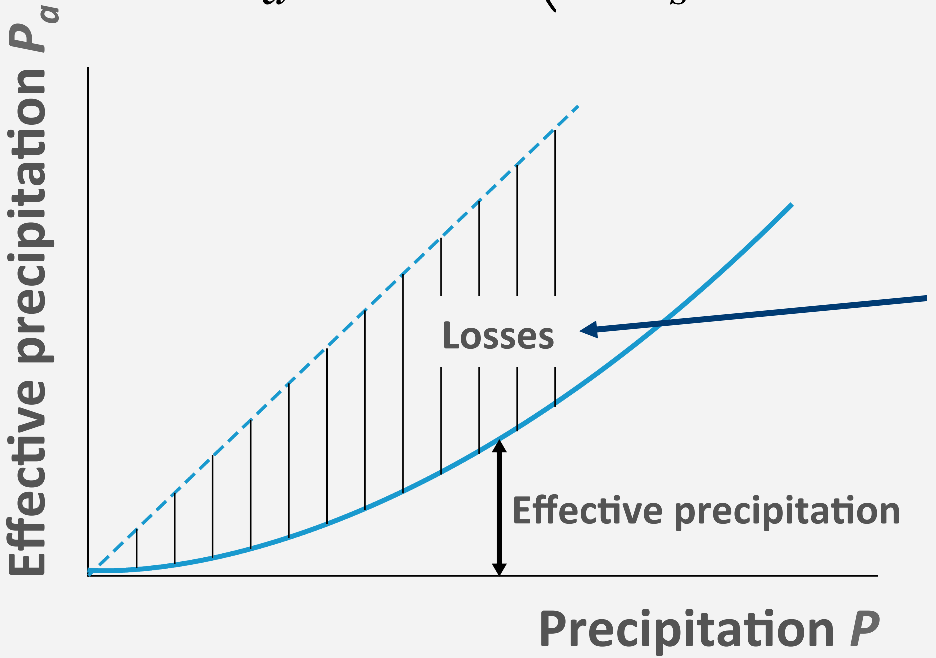
$$\int_t^\infty Q_t \,\mathrm{d}t = \int_t^\infty Q_0 \exp\left(-\frac{t}{k}\right) \mathrm{d}t = -k \left| Q_0 \exp\left(-\frac{t}{k}\right) \right|_t^\infty = -k\left(0 - Q_t\right) = kQ_t$$

Rainfall runoff processes

Effective precipitation

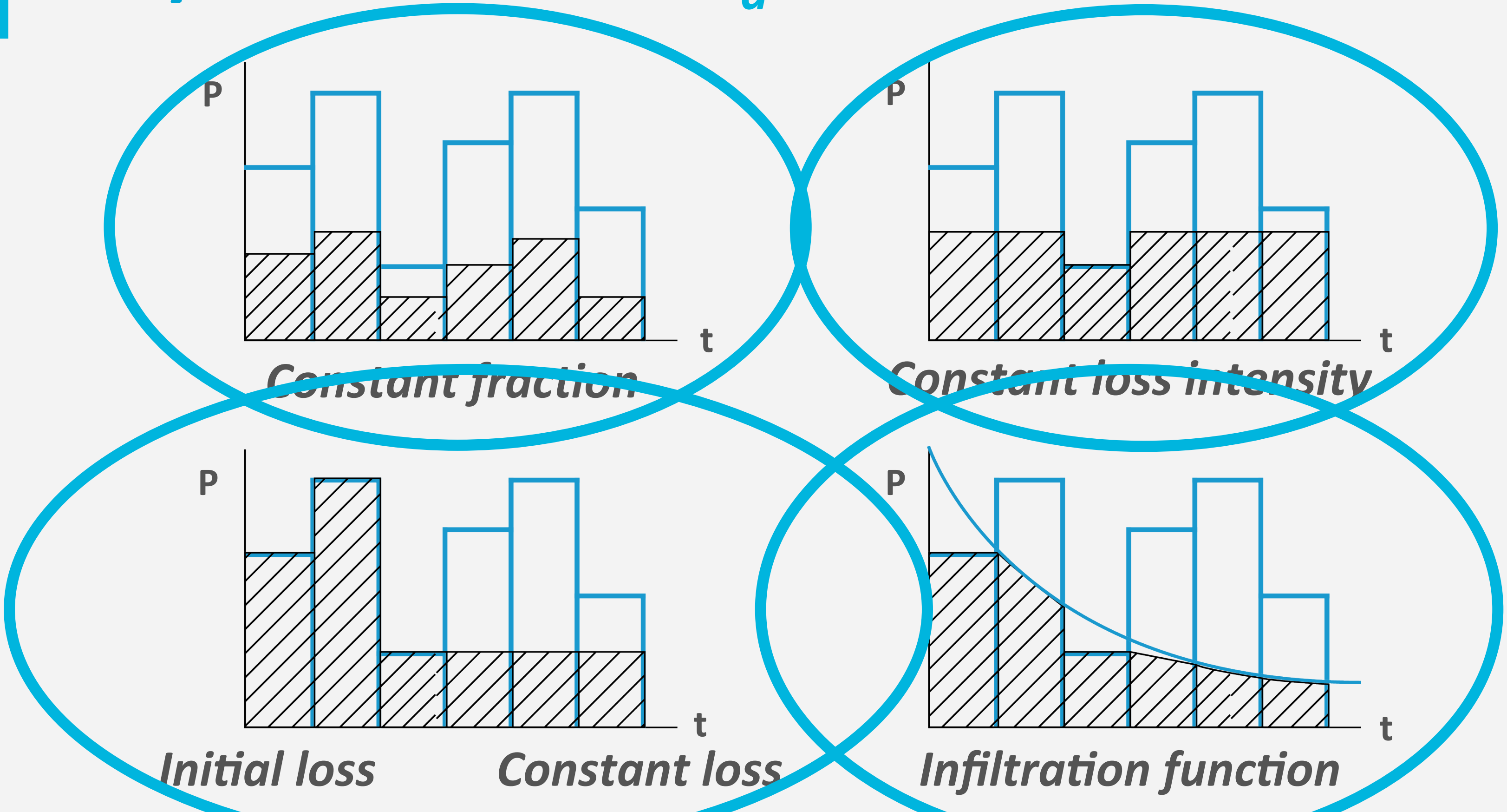
$$P_a = P - \left(\mathrm{d}S_s / \mathrm{d}t + \mathrm{d}S_u / \mathrm{d}t \right) - 0$$

0 (during event)



No 'losses', but pool formation and unsaturated storage increase, later to be evaporated (*I* and *T*)

Ways to determine P_a



The storage principle

$$S = kQ$$

Q discharge per unit surface area [L/T]

$$\frac{\mathrm{d}S}{\mathrm{d}t} = P_a - Q$$

hence:

$$k\frac{\mathrm{d}Q}{\mathrm{d}t} = P_a - Q$$

$$\frac{\mathrm{d}Q}{Q - P_a} = -\frac{1}{k}\mathrm{d}t$$

hence:

$$\ln\left(Q - P_a\right) = -\frac{t}{k} + C$$

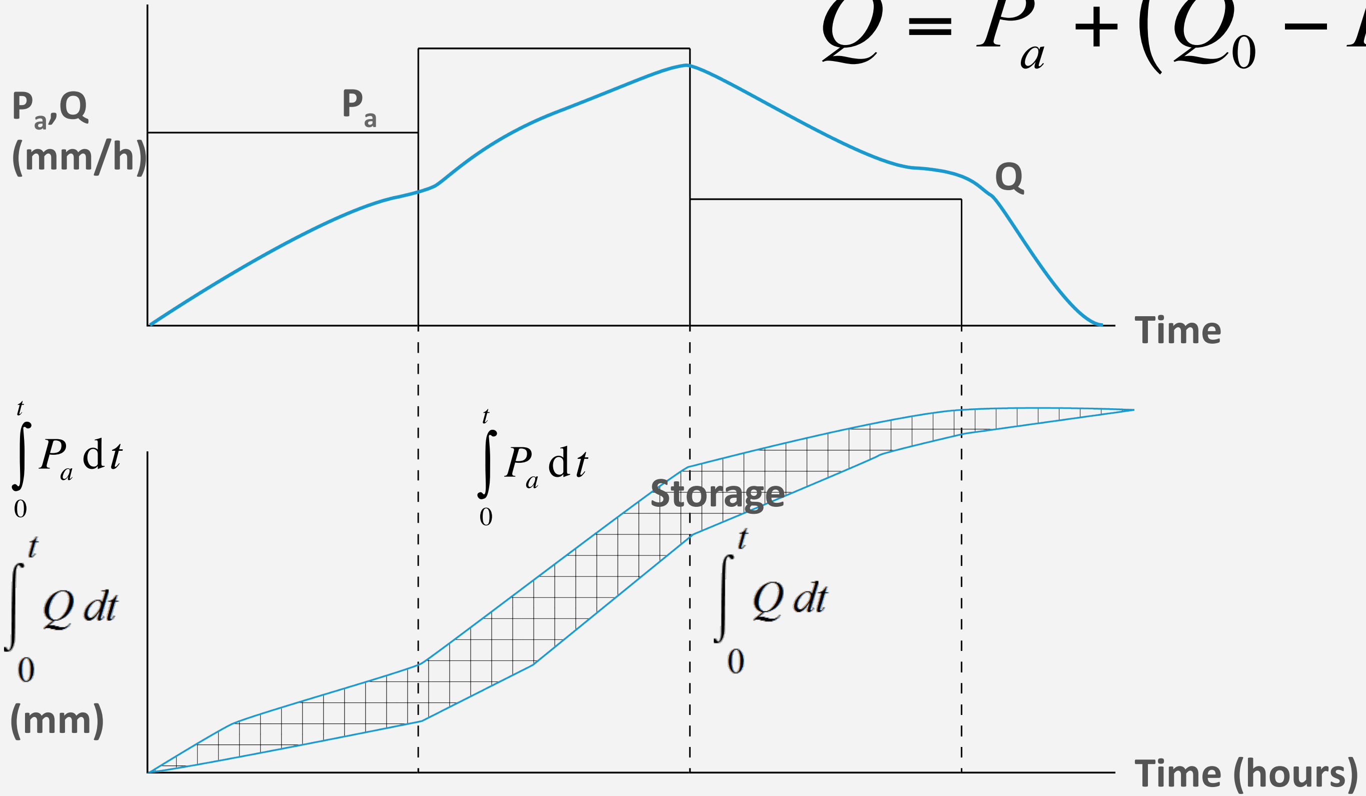
If t=0, then Q=Q_0

hence:

$$C = \ln\left(Q_0 - P_a\right)$$

The storage principle

$$Q = P_a + (Q_0 - P_a)\exp\left(-\frac{t}{k}\right)$$



The storage principle

Analytical:

$$Q = P_a + \left(Q_0 - P_a\right)\exp\left(-\frac{t}{k}\right)$$

Numerical:

$$\Delta S = \left(P_a - \bar{Q}\right)\Delta t$$

$$S_2 = S_1 + \left(P_a - \left(\frac{Q_1 + Q_2}{2}\right)\right)\Delta t$$

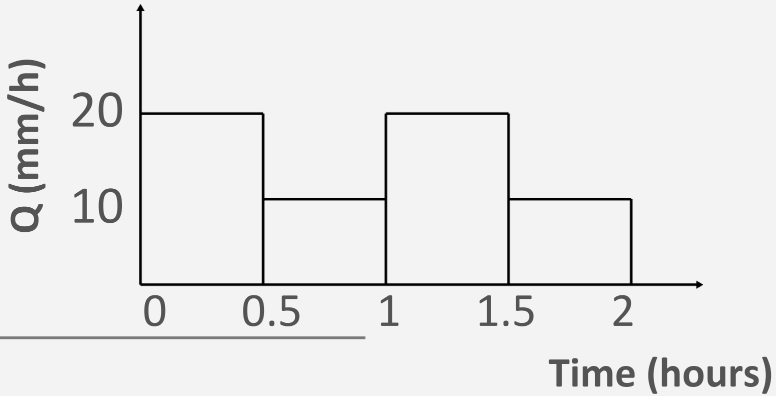
$S_1 = kQ_1$ ***and*** $S_2 = kQ_2$

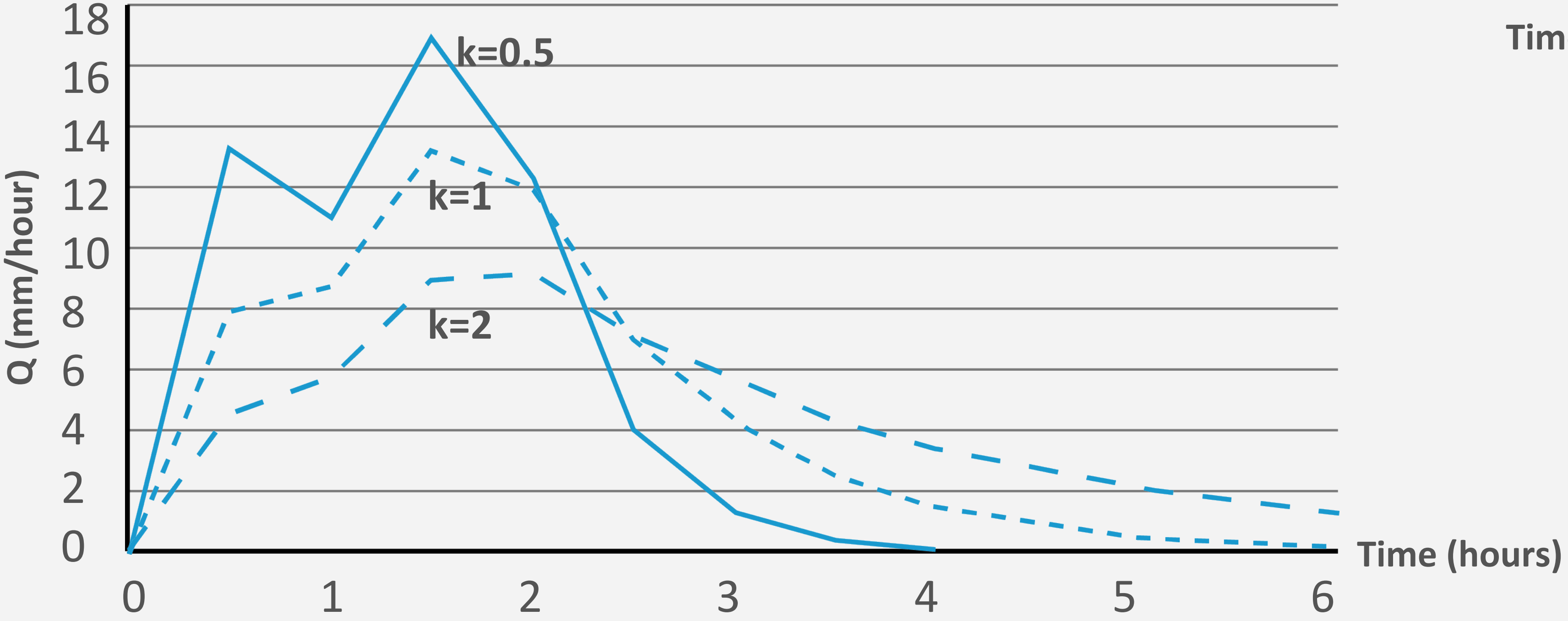
The storage principle

$$Q_2 = \frac{k - 0.5\Delta t}{k + 0.5\Delta t} Q_1 + \frac{\Delta t}{k + 0.5\Delta t} P_a$$

More appropriate for groundwater dominated catchments

Example storage principle

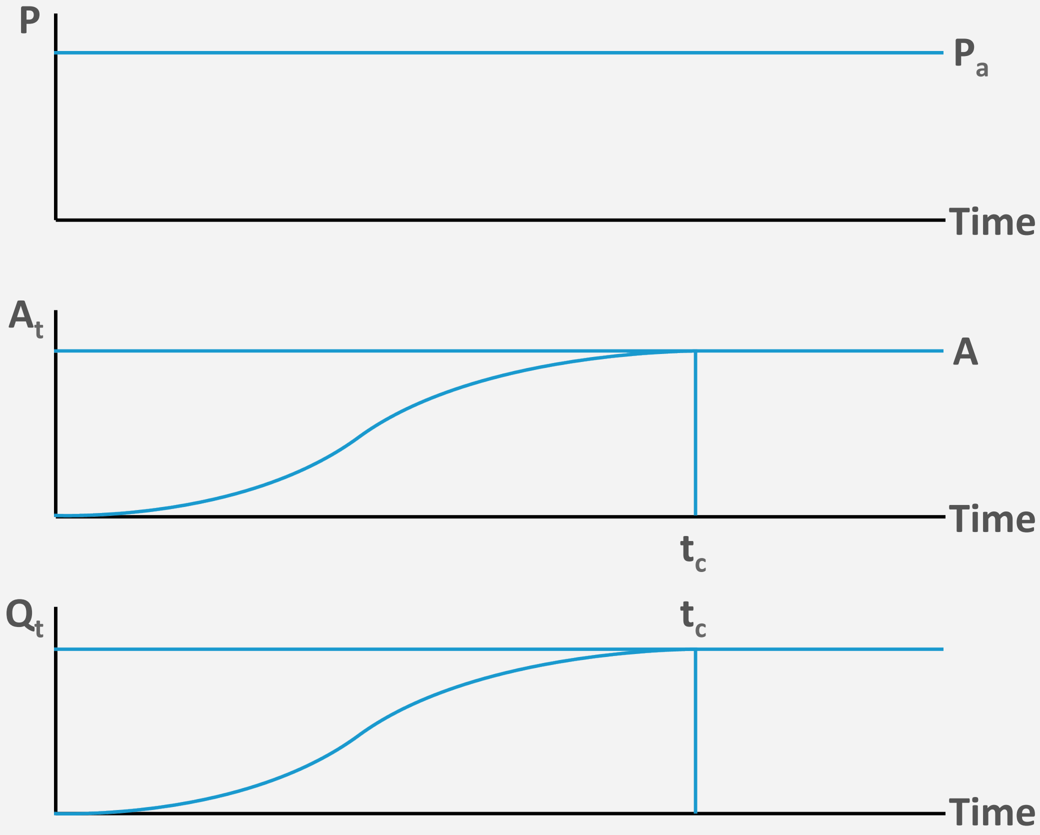


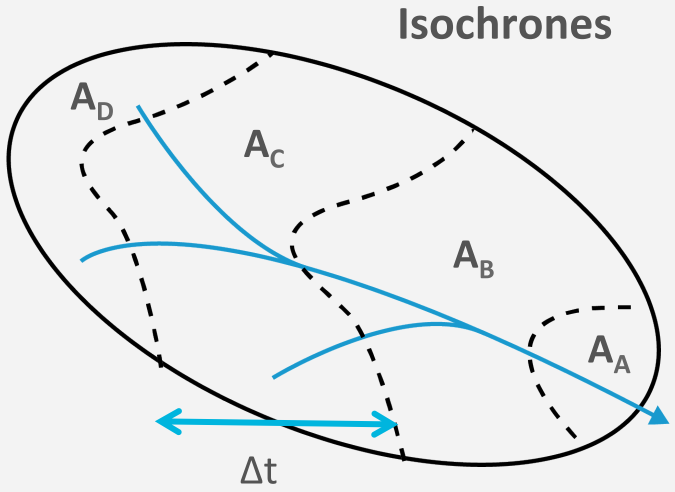


Runtime principle

- *Rational method*
- *Runoff is proportional to the contributing surface area, which increases in time*
- *Runoff is proportional to the effective precipitation, hence linearity*
- *Analogy with ping-pong balls*

Runtime principle





Assumptions of runtime principle

- *Runtime is time invariant (stationarity)*
- *Runtime is proportional to contributing area*
- *Linearity between Q and P_a*
- *Precipitation is equally distributed in space*
- *Often used in urban environments (impervious surfaces, urban drains, small impervious catchments)*

Runtime principle equations

$$Q(t) = P_a \min\left(A, A_t\right) = P_a A \min\left(1, \frac{A_t}{A}\right)$$

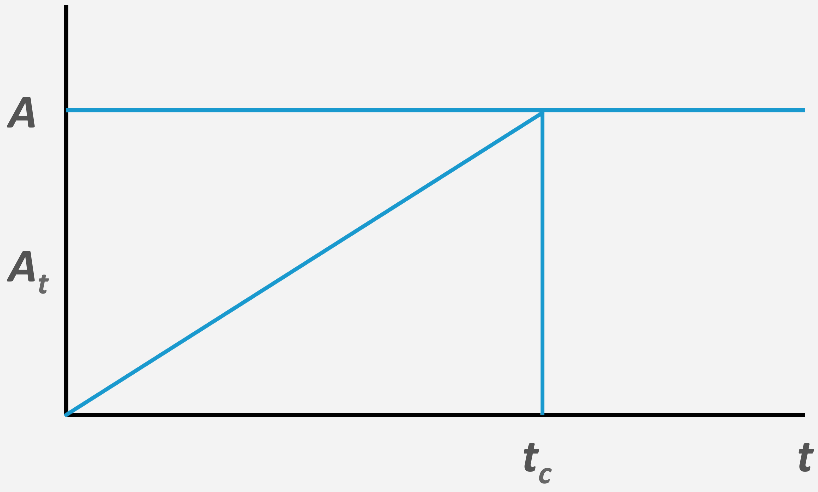
- *Q in [L^3T^{-1}], q in [LT^{-1}]*
- *P_a assumed constant over an interval*
- *When A_t equals A (total surface area of catchment), then $t=t_c$ (time of concentration)*
- *Superposition*

$$q(t) = \frac{Q(t)}{A} = P_a \min\left(1, \frac{A_t}{A}\right)$$

Runtime principle

Linear increase of A_t:

$$A_t = \frac{A}{t_c} t$$



$$q(t) = P_a \min\left(1, \frac{A_t}{A}\right) = P_a \min\left(1, \frac{t}{t_c}\right)$$

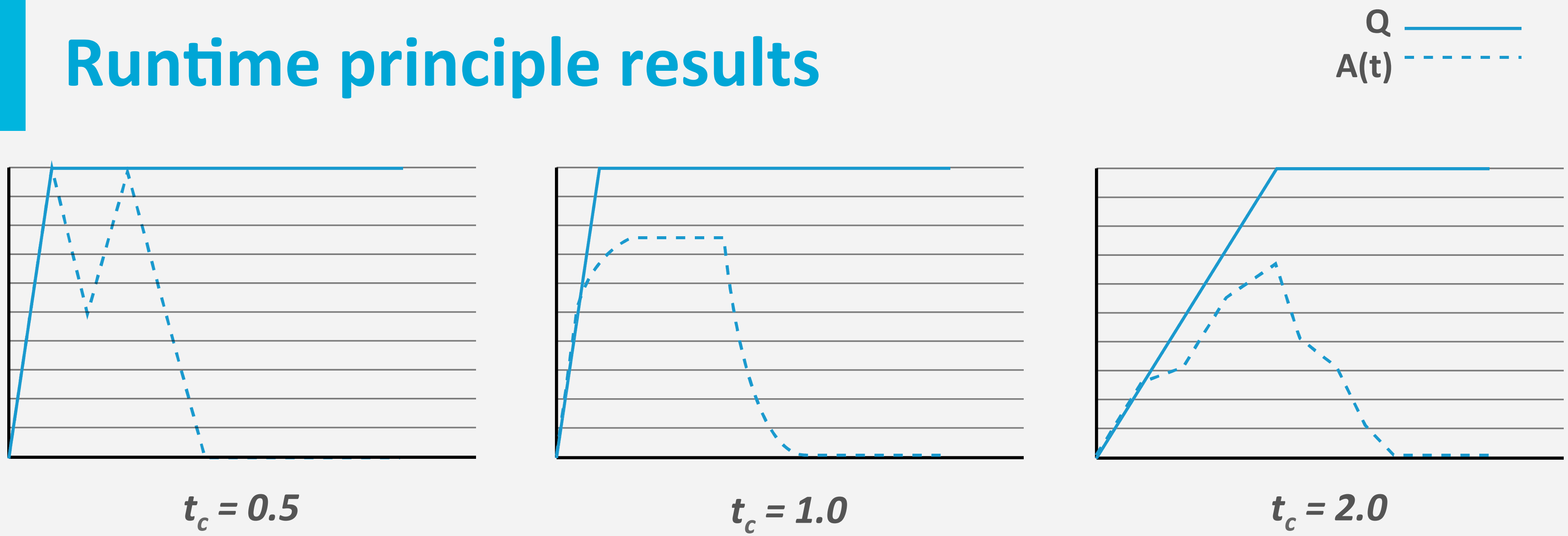
Runtime principle numerical equations

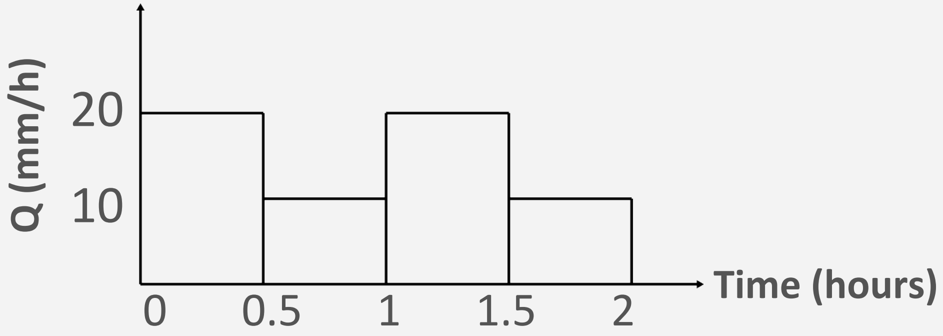
Superposition of subsequent events:

$$Q(t) = \sum_{i=1}^{n} \Delta P_{a,i} \cdot A \min\left(1, \frac{\max\left(A_{t-\Delta t(i-1)}, 0\right)}{A}\right)$$

$$q(t) = \sum_{i=1}^{n} \Delta P_{a,i} \min\left(1, \frac{\max\left(t - \Delta t(i-1), 0\right)}{t_c}\right)$$

Runtime principle results





Runtime principle questions

- *Why must the time step Δt be smaller than t_c?*
- *What happens if Δt is equal to t_c?*
- *Under which conditions can we apply the runtime principle?*

Applicability of the different methods

Storage principle

- ‘Flat areas’
- Groundwater dominated catchments

Runtime principle

- Steep impervious catchments
- Surface runoff

‘Real’ catchments

- Unit hydrograph
- Hydrological modelling

Challenge

- *This was treacherously simple*
- *Real catchments are more complex*
- *There still is a world to discover in hydrology*

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